Parallel Composition in a Paper by de Alfaro e.a. is not Associative

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Interface formalisms are used to model both input and output requirements of system components. In [dAdSF+05] de Alfaro e.a. present Sociable Interfaces in which interfaces can communicate via action synchronization as well as via shared variables. They show how interfaces can be composed via a product operator, and how bad states in the product can be pruned such that a parallel composition can be obtained. Moreover they show a refinement relation for sociable interfaces. The theory is implemented into the tool TICC, which is available from the website `http://dvlab.cse.ucsc.edu/Ticc`. We will show that their product operator on sociable interfaces is not an associative operator, contradicting the claim in the paper, and leading to an parallel operator that is not associative.

Given three modules M_1, M_2 and M_3, we show that $(M_1 \otimes M_2) \otimes M_3$ does not exist, whereas $M_1 \otimes (M_2 \otimes M_3)$ does. Hence we do not show non-associativity in a strong sense, since the example does not present a situation where a different order of composition really gives two existing and different systems. Nevertheless this weak form of non-associativity is unwanted.

We use all definitions, lemma's, and theorems from [dAdSF+05]. The signatures of the modules M_1, M_2 and M_3 look as follows:

$Act_1 = \{a, b\}$	$Act_2 = \{b\}$	$Act_3 = \{a\}$
$V_1^G = V_1^H = \{u\}$	$V_2^G = V_2^H = \{u\}$	$V_3^G = V_3^H = \emptyset$
$V_1^L = \{c_1\}$	$V_2^L = \{c_2\}$	$V_3^L = \{c_3\}$
$W_1(a) = \{c_1, u\}$		$W_3(a) = \{c_3\}$
$W_1(b) = \{c_1\}$	$W_2(b) = \{c_2, u\}$	

Clearly c_1, c_2, c_3 are local variables whereas u is the single global variable. There are two actions a, b on which synchronization is possible.

Now $M_1 \otimes (M_2 \otimes M_3)$ exists, but $(M_1 \otimes M_2) \otimes M_3$ does not, because $M_1 \otimes M_2$ does not exist. This is due to the fact that the product only exists if M_1 and M_2 are *composable* according to Definition 19. However $W_1(a) \cap V_2 = W_1(a) \cap (V_2^L \cup V_2^H) = \{u\} \neq \emptyset$, then the definition requires $a \in Act_2$ which is not the case.

We have shown non-associativity on the signatures of modules. In the following we show possible transition predicates, and thus modules with these kind of signatures indeed exist.

On `http://www.ita.cs.ru.nl/publications/papers/fvaan/BV07.html` the example can be found described in the language of the tool TICC. We assume all variables are booleans, now a possibility for the transition predicates is as follows:

$$\rho_1^O(a) = (c_1' = \neg c_1) \wedge (u' = \neg u)$$
$$\rho_1^O(b) = (c_1' = \neg c_1)$$
$$\rho_2^O(b) = (c_2' = \neg c_2) \wedge (u' = \neg u)$$
$$\rho_3^O(a) = (c_3' = \neg c_3)$$

References

[dAdSF+05] Luca de Alfaro, Leandro Dias da Silva, Marco Faella, Axel Legay, Pritam Roy, and Maria Sorea. Sociable interfaces. In Bernhard Gramlich, editor, *FroCos*, volume 3717 of *Lecture Notes in Computer Science*, pages 81–105. Springer, 2005.