New Fault Domains for Conformance Testing of Finite State Machines

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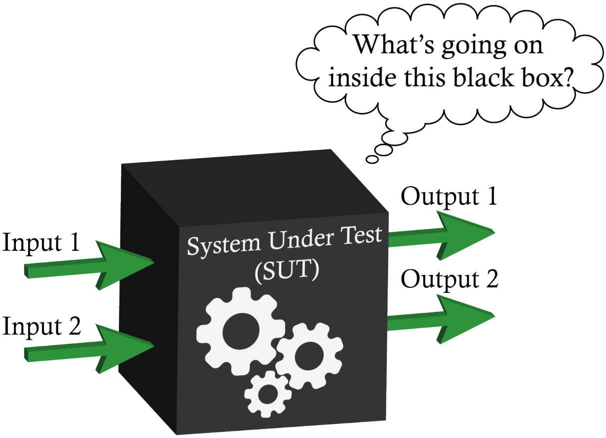
Black-box Conformance Testing



Question: Does SUT conform to its Spec?

Conformance Testing of FSMs: A Classic Problem

- Idea of (black-box) conformance testing can be traced back to Moore (1956) and Hennie (1964)
- Seminal results by Vasilevski (1973) and Chow (1978)
- Influential survey paper by Lee & Yannakakis (1994)
- Continued progress by model-based testing community

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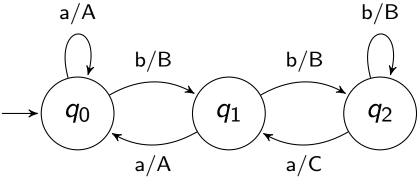
- Continued progress by model-based testing community
- Recent work on model learning poses major new challenges but also brings major new opportunities

Mealy Machines

The diagram below shows a simple Mealy machine with:

- finite sets of inputs $I = \{a, b\}$ and outputs $O = \{A, B, C\}$
- finite set of states $Q = \{q_0, q_1, q_2\}$ and initial state q_0
- transition function $\delta : Q \times I \to Q$
- output function $\lambda : Q \times I \to O$

We assume all states are reachable from the initial state.



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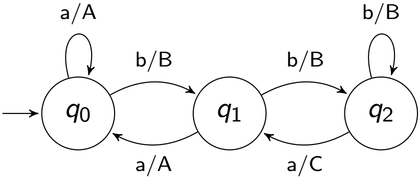

Mealy machines $\mathcal{M}$ and $\mathcal{N}$ are equivalent, $\mathcal{M} \approx \mathcal{N}$, iff for every sequence of inputs they produce the same sequence of outputs.

Conformance Testing

Definition (Conformance Testing)

Let $\mathcal{S}$ be a Mealy machine (the specification).

- A sequence $\sigma \in I^*$ is called a test for $\mathcal{S}$.
- Mealy machine $\mathcal{M}$ passes test σ for $\mathcal{S}$ iff $\mathcal{M}$ and $\mathcal{S}$ produce the same outputs in response to input sequence σ.
- A test suite T for $\mathcal{S}$ is a finite set of tests for $\mathcal{S}$.
- $\mathcal{M}$ passes T iff it passes all tests in T.

Fault Domains

A fault domain reflects assumptions about faults that may occur in an implementation and that need to be detected during testing:

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Definition (Fault domains and $\mathcal{U}$-completeness)

Let $\mathcal{S}$ be a Mealy machine. A fault domain is a set $\mathcal{U}$ of Mealy machines. A test suite T for $\mathcal{S}$ is $\mathcal{U}$-complete if, for each $\mathcal{M} \in \mathcal{U}$, $\mathcal{M}$ passes T implies $\mathcal{M} \approx \mathcal{S}$.

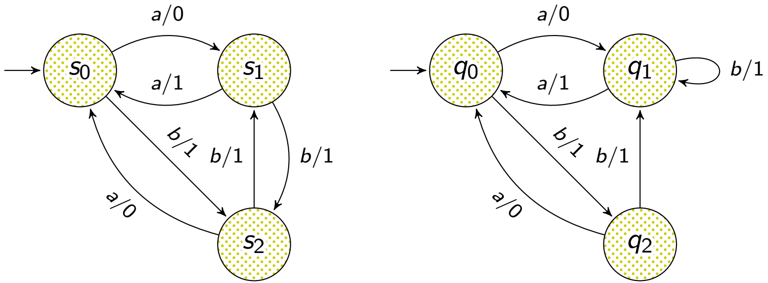
The Most Popular Fault Domain Ever?

Based on work of Moore, Hennie, and Chow, hundreds of papers about conformance testing use the following fault domain:

Definition ($\mathcal{U}_m$)

For $m > 0$, $\mathcal{U}_m$ is the set of Mealy machines with at most m states.

Example



Test suite $T = \{aaaa, abaa, baaa, bbaa\}$ for specification $\mathcal{S}$ on the left is $\mathcal{U}_3$-complete. SUT $\mathcal{M}$ on the right does not pass test $abaa$.

The Problem with $\mathcal{U}_m$

1. The size of $\mathcal{U}_m$-complete test suites grows exponentially in $m - n$, where n is the number of states of $\mathcal{S}$
2. Thus we can only run $\mathcal{U}_m$-complete test suites for small values of $m - n$ (say 2 or 3)
3. But the assumption that faults introduce at most a few extra states is not realistic

A Better Fault Domain

Definition ($\mathcal{U}_k^A$)

Let k be a natural number and let $A \subseteq I^*$. Then $\mathcal{U}_k^A$ is the set of all Mealy machines $\mathcal{M}$ such that every state of $\mathcal{M}$ can be reached by an input sequence $\sigma\rho$, for some $\sigma \in A$ and $\rho \in I^{\leq k}$.

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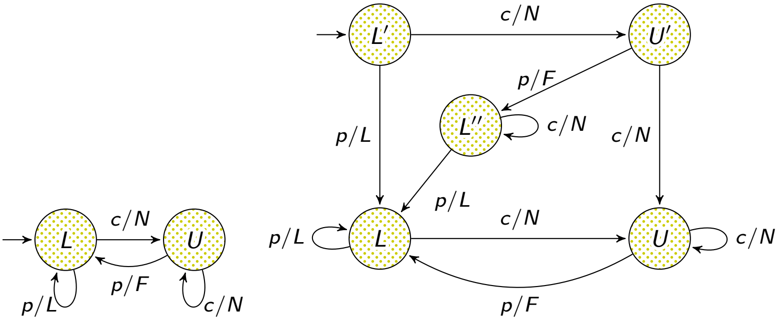
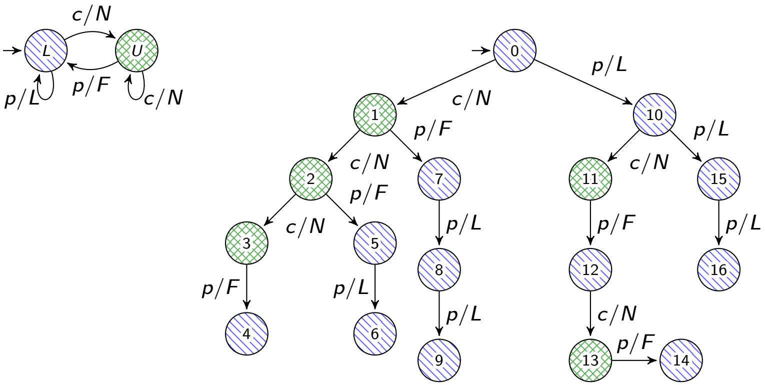
Intuition: set A describes some required behaviors, e.g. the happy flow of a protocol. An implementor will aim to support these behaviors, but may make mistakes along the way.

Turnstile Example



Mealy machine on the right is contained in fault domain $\mathcal{U}_1^A$, for $A = \{\epsilon, c\}$, since all states can be reached via at most one transition from states L' and U' that are reachable via A.

SPYH method is not $\mathcal{U}_1^A$-complete



Test suite $T = \{cccp, ccpp, cppp, pcpcp, ppp\}$, represented here as a tree, was generated using SPYH-method and is $\mathcal{U}_3$-complete for machine on the left. Test suite is not $\mathcal{U}_1^{\{\epsilon,c\}}$-complete.

A Better Fault Domain (cnt)

The number of states of FSMs in $\mathcal{U}_k^A$ grows exponentially in k:

Lemma

Let $A \subset I^*$ be prefix closed with $|A| = n$, let $|I| = l$ and $k > 0$. Then fault domain $\mathcal{U}_k^A$ contains Mealy machines with up to $n + (\sum_{j=0}^{k-1} l^j)(nl - n + 1)$ states.

Relation between $\mathcal{U}_m$ and $\mathcal{U}_k^A$

Let A be a minimal state cover for a minimal specification $\mathcal{S}$. $\mathcal{U}_m$-complete test suites typically check whether distinct sequences from A lead to distinct states in the SUT:

Definition ($\mathcal{U}^A$)

Let $A \subseteq I^*$. Then $\mathcal{U}^A$ is the set of all Mealy machines $\mathcal{M}$ such that there are $\sigma, \rho \in A$ with $\sigma \neq \rho$ and $\delta^{\mathcal{M}}(q_0^{\mathcal{M}}, \sigma) \approx \delta^{\mathcal{M}}(q_0^{\mathcal{M}}, \rho)$.

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Theorem

Let $A \subset I^*$ be a finite set of input sequences with $\epsilon \in A$. Let k and m be natural numbers with $m = |A| + k$. Then $\mathcal{U}_m \subset \mathcal{U}_k^A \cup \mathcal{U}^A$.

A Better Fault Domain (cnt)

Let A be a set of traces describing the happy flow of TLS protocol. We computed the eccentricity of SUT models from De Ruiter et al:[1]

SUT model client	States	Eccentricity	SUT model server	States	Eccentricity
GnuTLS 3.3.12 full	9	0	GnuTLS 3.3.12 full	9	0
GnuTLS 3.3.8 full	15	1	GnuTLS 3.3.12 regular	7	0
GnuTLS 3.3.8 regular	11	1	GnuTLS 3.3.8 full	15	1
NSS 3.17.4 full	11	0	GnuTLS 3.3.8 regular	12	1
NSS 3.17.4 regular	7	0	NSS 3.17.4 regular	8	1
OpenSSL 1.0.1g regular	10	3	OpenSSL 1.0.1j regular	11	3
OpenSSL 1.0.1j regular	6	0	OpenSSL 1.0.1l regular	10	3
OpenSSL 1.0.1l regular	6	0	OpenSSL 1.0.2 regular	9	1
OpenSSL 1.0.2 full	8	0	RSA BSAFE C 4.0.4 regular	9	1
OpenSSL 1.0.2 regular	6	0	RSA BSAFE Java 6.1.1 regular	6	0
			miTLS 0.1.3 server regular	6	0

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Indication that fault domains $\mathcal{U}_k^A$ may be practically relevant.

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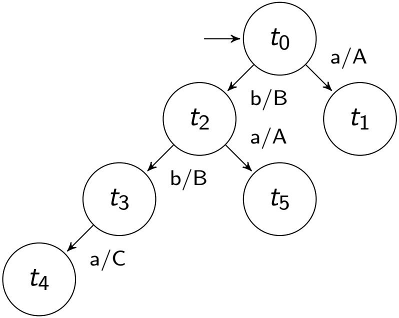
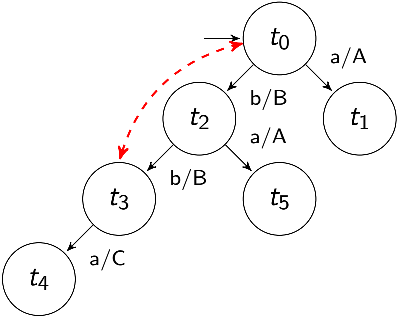
Apartness



Definition (Apartness)

Two states t, u in a testing tree are apart, $t \# u$, if they both enable an input sequence $\sigma \in I^*$ for which the outputs are different.

Apartness



$a \vdash t_3 \# t_0$

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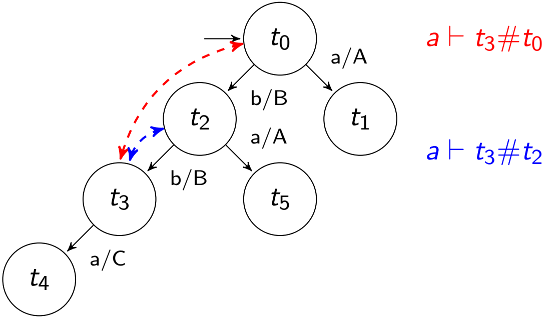

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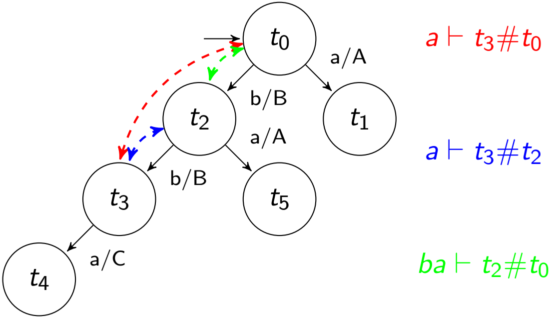

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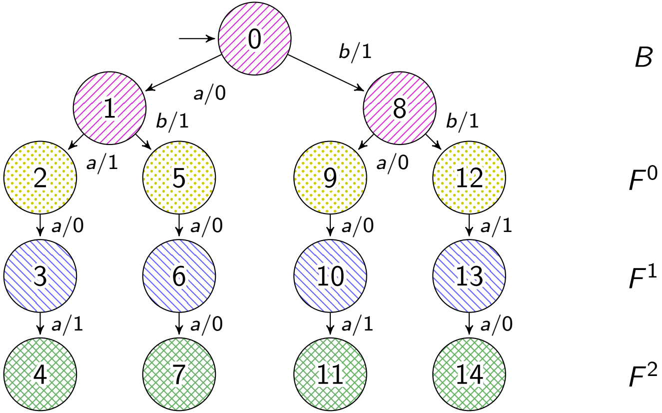
Basis

Definition (Basis)

Let $\mathcal{T}$ be a testing tree. A nonempty subset of states $B \subseteq Q^{\mathcal{T}}$ is called a basis of $\mathcal{T}$ if B is ancestor-closed and all states in B are pairwise apart.
For each state q of $\mathcal{T}$, the candidate set $C(q)$ is the set of basis states that are not apart from q: $C(q) = \{q' \in B \mid \neg(q \# q')\}$.
A state q of $\mathcal{T}$ is identified if its candidate set is a singleton.

Example


0
b/1
a/0
1
8
b/1
b/1
2
5
9
12
a/1
a/0
a/0
a/0
a/0
a/1
3
6
10
13
a/1
a/0
a/1
a/0
4
7
11
14
B
F^0
F^1
F^2

Stratification

Definition (Stratification)

A basis B induces a stratification of $Q^{\mathcal{T}}$ as follows:

1. We write F^0 for the set of immediate successors of basis states that are not basis states themselves.
2. For $k > 0$, F^k is the set of immediate successors of F^{k-1}.

For $k \in \mathbb{N}$, we write $F^{<k} = \bigcup_{i<k} F^i$.

A Sufficient Condition for k-A-Completeness

Theorem

Let $\mathcal{S}$ be a Mealy machine, let T be a test suite for $\mathcal{S}$, let $\mathcal{T} = \mathrm{Tree}(\mathcal{S}, T)$, let B be a basis for $\mathcal{T}$ with $|B| = |Q^{\mathcal{S}}|$, let $A = \mathrm{access}(B)$, let $F^0, F^1, \ldots$ be the stratification induced by B, and let $k \geq 0$. Suppose all states in $B \cup F^{<k}$ are complete, all states in F^k are identified, and:

$$\forall q \in F^k \; \forall r \in F^{<k} : \qquad C(q) = C(r) \vee q \# r$$

Then T is k-A-complete.

Complexity

Theorem

Let $\mathcal{S}$ be a Mealy machine and let T be a test suite for $\mathcal{S}$. Let N be the number of states in the tree representation of T. Then there is a $O(N^2)$ algorithm that checks whether the testing tree for T satisfies our sufficient condition for k-A-completeness.

k-A-Completeness Existing Methods

Corollary

The Wp, HSI, W, UIOv, ADS and hybrid ADS methods generate k-A-complete test suites.

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Theorem

The SPYH, SPY and H methods do not ensure k-A-completeness.

Conclusions

1. We proposed new fault domains, leading to the notion of *k-A-completeness*, which may be of practical interest since the number of extra states grows exponentially in k.

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3. Our condition implies k-A-completeness of several existing test suite generation approaches, e.g. the Wp and HSI methods.
4. Counterexamples show that H, SPY and SPYH methods do not guarantee k-A-completeness.

Future Work

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3. Develop efficient test suite generation algorithms based on our characterization
4. Bridge the gap between sensible fault domains and practical test generation methods