Properties of P^2TA

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Priced Probabilistic Timed Automata

Definition

A Priced Probabilistic Timed Automaton consists of

- L, finite set of locations
- $l_{init} \in L$, initial location
- $\mathbb{X}$, finite set of clocks
- $pE \subseteq L \times Zones(\mathbb{X}) \times 2^{\mathbb{X}} \times Dist(L)$, probabilistic edges
- $\dot{\$} : L \longrightarrow \mathbb{N}$, price rate for each location

Problem

Timed cost-bounded probabilistic reachability ✔ $\frac{}{2}$

Cost-bounded Probabilistic Reachability – Maximum

Problem

Is it true that...?

- *one can reach state G*
- *within time $\leq t$*
- *with cost $\leq \kappa$*
- *with probability $> p$*
- *under the best scheduler*

This problem has been solved partially: There is a (semi-decidable) algorithm. [Berendsen/Jansen/Katoen, QEST 2006]

Cost-Bounded Probabilistic Reachability – Minimum

Problem

Is it true that...?

- *one cannot avoid state G*
- *within time $\leq t$*
- *with cost $\leq \kappa$*
- *with probability $\geq p$*
- *under the worst scheduler*

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- $\dot{\$} : L \to \mathbb{N}$, price rate for each location
- $inv : L \to Zones(\mathbb{X})$, location invariants

Problem

Timed cost-bounded probabilistic worst-case reachability ?

Reformulation

Given a scheduler A,

$$
\begin{aligned}
Prob^A(\diamond(\Phi \wedge cost \leq k)) = \\
&= 1 - Prob^A(\neg\Phi \,\mathcal{W}\, cost > k) = \\
&= 1 - Prob^A(\neg\Phi \,\mathcal{U}\, BSCC(\neg\Phi) \vee cost > k)
\end{aligned}
$$

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\min_A Prob^A(\diamond(\Phi \wedge cost \leq k)) = \\
&= 1 - \max_A Prob^A(\neg\Phi \, \mathcal{U} \, BSCC(\neg\Phi) \vee cost > k)
\end{aligned}
$$